



# Contest Quiz 2

## Question Sheet

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In this quiz we will review concepts of probability and statistics covered in the first three lectures.

NOTE: Please round your results to *two decimal places*. Do *not* round any interim results.

EXAMPLE: If your unrounded solution is 0.13897439, drop all decimal places except the first three. This leaves you with 0.138. If the third decimal place is 5 or above (as is the case here), round up. This gives 0.14.

### Question 1

Let the number of “tails” in a toss of two fair coins be denoted by  $X$ .

- (i) What is the probability that  $X = 1$ ?
- (ii) What is the probability that  $1 \leq X \leq 2$ ?
- (iii) What is the mean of  $X$ ?
- (iv) What is the variance of  $X$ ?

### Question 2

The credit risk manager of a bank is interested in the relationship between loan default and firm size. She has the following information on the joint probability distribution between the two binary variables.

	Default ( $Y = 0$ )	Repayment ( $Y = 1$ )	Total
Small firms ( $X = 0$ )	0.045	0.709	0.754
Large firms ( $X = 1$ )	0.005	0.241	0.246
Total	0.050	0.950	1.000

- (i) Compute  $E(Y)$ .
- (ii) Calculate  $E(Y|X = 0)$ .
- (iii) Calculate the default rate for small firms. Hint: Use the fact that Default Rate =  $\frac{\text{No. defaults}}{\text{No. total loans}} = Pr(Y = 0) = 0.050 = 1 - 0.950 = 1 - E(Y)$ .
- (iv) A delinquent firm ( $Y = 0$ ) is selected at random from this population. What is the probability that this firm is large ( $X = 1$ )?

### Question 3

Compute the following probabilities.

- (i) If  $X \sim N(1, 4)$ , find  $Pr(X \leq 3)$ .
- (ii) If  $X \sim N(3, 9)$ , find  $Pr(X > 0)$ .
- (iii) If  $X \sim N(50, 25)$ , find  $Pr(40 \leq X \leq 52)$
- (iv) If  $X \sim N(5, 2)$ , find  $Pr(6 \leq X \leq 8)$

### Question 4

In a population  $\mu_X = 100$  and  $\sigma_X^2 = 43$ . Use the central limit theorem to answer the following questions:

- (i) In a random sample of size  $n = 100$ , find  $Pr(\bar{X} \leq 101)$ .
- (ii) In a random sample of size  $n = 165$ , find  $Pr(\bar{X} > 98)$ .
- (iii) In a random sample of size  $n = 64$ , find  $Pr(101 \leq \bar{X} \leq 103)$ .

### Question 5

Suppose  $X_i, i = 1, 2, \dots, n$ , are i.i.d. random variables, each distributed  $N(10, 4)$ . Compute  $Pr(9.6 \leq \bar{X} \leq 10.4)$  when

- (i)  $n = 20$
- (ii)  $n = 100$
- (iii)  $n = 1,000$

### Question 6

Every year, Indian farmers are exposed to the risk of losing their crop due to severe droughts. From year to year, the damage due to drought is random. Denote by  $Y$  the value in GBP of damage due to drought in any given year. Suppose that in 95% of the years  $Y = 0\text{GBP}$ , but in 5% of the years  $Y = 20,000\text{GBP}$ .

- (i) What is the mean of the damage due to drought in any year?
- (ii) What is the standard deviation of the damage due to drought in any year?

Now consider a risk sharing arrangement of 100 farmers whose fields are sufficiently dispersed so that, in any year, the damage to different fields can be assumed to be independently distributed random variables. Denote by  $\bar{Y}$  the average damage due to drought to these 100 fields in a year.

- (iii) What is the expected value of the average damage due to drought,  $\bar{Y}$ ?
- (iv) What is the probability that  $\bar{Y}$  exceeds 2,000GBP?

## Question 7

A car manufacturer has to decide on the style of a product feature of his new truck. The consulting company in charge of the market research conducts a representative customer survey. The results are as follows

<i>Style A</i> preferred	<i>Style B</i> preferred	Total respondents
215	185	400

Let  $p$  denote the fraction of *all likely customers* who preferred *Style A*, and let  $\hat{p}$  be the fraction of *survey respondents* who preferred *Style A*.

- Use the results of the market research to estimate  $p$ . Hint: Each customer's preference can be denoted by  $Y$ .  $Y = 1$  if the customer prefers *Style A* and  $Y = 0$  if the customer prefers *Style B*.  $Y$  is a Bernoulli random variable with probability  $\Pr(Y = 1) = p$  and  $\Pr(Y = 0) = 1 - p$ .  $Y$  has mean  $p$  and variance  $p(1 - p)$ .
- Use the estimator of the variance of  $\hat{p}$ ,  $\hat{p}(1 - \hat{p})/n$ , to calculate the standard error of your estimator.
- What is the  $p$ -value for the test  $H_0 : p = 0.5$  vs.  $H_a : p \neq 0.5$ ? Hint: because of the large sample size ( $n = 400$ ) you may use the Standard Normal distribution to get the  $p$ -value.
- What is the  $p$ -value for the test  $H_0 : p = 0.5$  vs.  $H_a : p > 0.5$ ?
- Did the market research contain statistically significant (5%-level) evidence that *Style A* is preferred to *Style B*?
  - yes
  - no

## Question 8

Using the data in Question 7:

- Construct a 95% confidence interval for  $p$  and give the lower bound.
- Construct a 99% confidence interval for  $p$  and give the upper bound.
- Without doing any additional calculations, test the hypothesis  $H_0 : p = 0.5$  vs.  $H_a : p \neq 0.5$  at the 5% significance level. Can you reject the null hypothesis?
  - yes
  - no