Logarithmic Models

Logarithmic Models - linear-log

$$Y_i = \beta_0 + \beta_1 \cdot lnX_i + u_i$$

marginal effects:

$$\begin{aligned} \frac{\partial Y_i}{\partial X_i} &= \beta_1 \frac{1}{X_i} \\ \Delta Y_i &\approx \Delta X_i \cdot \beta_1 \frac{1}{X_i} \end{aligned}$$

if X_i changes by 1% ($\Delta X_i = 0.01 \cdot X_i$) ...

$$\Delta Y_i \approx 0.01 X_i \cdot \beta_1 \frac{1}{X_i}$$

... Y_i changes by $0.01 \cdot \beta_1$.

Logarithmic Models - log-linear

$$lnY_i = \beta_0 + \beta_1 \cdot X_i + u_i$$

marginal effects:

$$\begin{aligned} \frac{\partial lnY_i}{\partial X_i} &= \beta_1 \\ \frac{\Delta lnY_i}{\Delta X_i} &\approx \beta_1 \end{aligned}$$

and

$$\frac{\Delta Y_i}{Y_i} \approx \Delta lnY_i \approx \beta_1 \cdot \Delta X_i$$

A change of X_i by one unit translates into a change of Y_i by the share β_1.

Logarithmic Models - log-log

$$\begin{aligned} lnY_i &= \beta_0 + \beta_1 \cdot lnX_i + u_i \\ Y_i &= e^{\beta_0} \cdot X_i^{\beta_1} \cdot e^{u_i} \end{aligned}$$

marginal effects:

$$\begin{aligned} \frac{\partial Y_i}{\partial X_i} &= e^{\beta_0} \cdot \beta_1 X_i^{\beta_1 - 1} = \beta_1 \frac{Y_i}{X_i} \\ \frac{\partial Y_i}{\partial X_i} \cdot \frac{X_i}{Y_i} &= \beta_1 \end{aligned}$$

β_1 is the elasticity of Y_i with respect to X_i.